LOAN RESTUCTURING ANALYSIS

TRANSFER PRICING GUIDE

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List of Abbreviations

The following abbreviations and symbols are used in this guide:

AAF	Annuity adjustment factor
FMV	Fair market value
IPA	Interest Prepayment Agreement
PIK	Pay-in-kind
NPV	Net present value
OECD Guidelines	“BEPS Actions 8 – 10, Financial Transactions”, a draft published in July – September 2018 for the purposes of public discussion
OID	Original issue discount
QED	Quod Erat Demonstrandum (that which was to be demonstrated)
SWPM	Bloomberg interest rate swap tool
TP	Transfer pricing
WACC	Weighted-average cost of capital
WHT	Withholding tax
YTM	Yield to maturity

Section 1 Introduction

Loan restructuring problem refers to the situation when the cash flows under the original loan agreement are restructured into new cash flows. The agreement does not provide provisions for new restructured cash flows and both the lender and the borrower need to agree to the modified terms of the agreement.

Loan restructuring typically occurs in two situations.

1. **Loan unwinding**. The borrower wants to repay (lend wants to request loan repayment) the loan partially or fully but the prepayment (pay-on-demand) provision is not included in the loan agreement.
2. **Liquidity management**. The borrower would like to defer or prepay early interest on the loan due to insufficient or excess cash balances.

Loan restructuring problem can generally broken-down into the following components.

1. Discount **rates and factors estimation**. There are three alternative approaches to discount rate calculation.
   - **Term structure of refinancing rates**. This is the recommended approach. Using refinancing rates ensures that the restructured cash flows are priced based on the bond market prices. Applying the refinancing rate term structure ensures that the discount rates with correct maturity terms are applied to each cash flow payment.
   - **Fixed refinancing rate**. This is a simplified approach. It is still based on the bond market prices but does not take into account the term of each cash flow payment. The approach can be recommended if a single loan is being restructured and the discount rate is estimated based on the loan maturity term. If multiple loans with different maturities are restructured, then it is not clear which maturity term to use for discount rate calculations and application of constant refinancing rate is inconsistent for different loans.
   - **WACC or risk-free rate**. The WACC rate is not based on the bond market prices and takes into account also the borrower-specific information (such as the borrower capital structure or usage of the repaid funds). Since WACC is reported as a single number, it can be applied to multiple loan instruments. Alternatively, a risk-free rate can be applied as discount rate (as for example is applied in Bloomberg's SWPM interest rate swap valuation tool). When the borrower and lender-specific facts are taken into account, a different discount rate may be applied to the borrower and the lender (due to differences in valuation approach which deviates from the valuation approach based on market loans / notes prices). Analysis of loan restructuring problem with the discrepancy in the lender and borrower discount rates is discussed in Section 4.
2. **Cash flow estimation**. The cash flows are constructed for the original and restructured loans. Depending on the terms of the loan, the cash flows for the original loan may be estimated under different scenarios (e.g. in the case of loan prepayment provision, the loan may be estimated assuming loan early prepayment or assuming loan is held to the maturity).
3. **NPV valuation**. The NPV valuation is performed as discussed in the related NPV Valuation guide. The terms of loan restructuring are selected so that both the borrower and the lender are better off from the restructured loan, where the cost/gain of the borrower and lender are estimated based on the NPV calculations.
4. **Proxy fair market valuation**. The NPV calculations produce the valuation numbers which may not be easy to interpret directly. Proxy FMV is discussed to show how the results can be explained consistently with economic intuition. Proxy calculations are performed by decomposing the loan restructuring into basic restricting steps. For example, interest prepayment can be decomposed

into the sum of each coupon prepayment (which can be approximately assessed without performing NPV calculations).[1]

Loan restructuring valuation can be performed under different assumptions.

1. The 'standard' loan restructuring analysis refers to the analysis performed under the following assumptions.
   - ► The same discount rate is applied to both the borrower and the lender. The assumption ensures existence of the restructured loan terms which makes both the borrower and lender indifferent between the original and restructured loans.
   - ► The analysis is performed as **before-tax** analysis without taking into consideration the tax impact on the borrower and lender assessment of loan restructuring problem.
2. In Section 4 we discuss the loan restructuring problem assuming that the discount rate applied to the borrower and lender cash flows is different. We analyze the relationship between the discount rate differential and existence of solutions to the loan restructuring problem.
3. Finally, in Section 5 we discuss the **tax analysis** of a loan restructuring problem.

[1] Further details and examples are provided in Section 3.

Section 2 Definitions and Examples

This section presents loan valuation analysis under different scenarios of loan cash flows prepayment or deferral structures. The section shows a conceptual difference between the cash flows prepayment and deferral structures and derives conditions under which both the borrower and the lender gain from the restructured loan.

2.1 Definitions

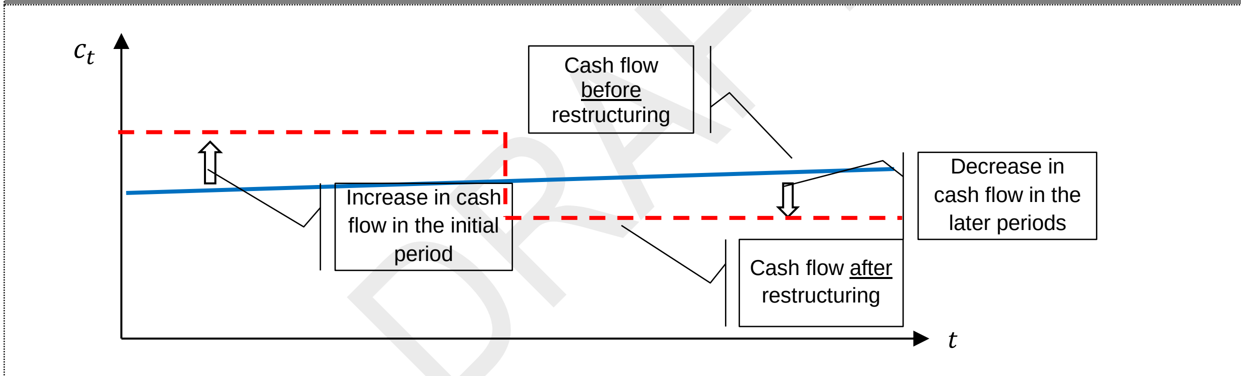
The loan prepayment and loan with deferred payments are defined as follows.

Definition [loan prepayment]: Suppose that $c_1, \dots, c_T$ are cash flows before the restructuring and $\tilde{c}_1, \dots, \tilde{c}_T$ are cash flows after the restructuring. Suppose also that $C_t = \sum_{s \le t} c_s$ and $\tilde{C}_t = \sum_{s \le t} \tilde{c}_s$ denote cumulative cash flows before and after loan restructuring. Then the loan restructuring transaction is called to be the loan **prepayment** transaction if

$$C_t \geq \tilde{C}_t \text{ for each } t \text{ and } C_T = \tilde{C}_T$$

Example of loan prepayment restructuring transaction is illustrated in the diagram below.

Loan prepayment type of transaction

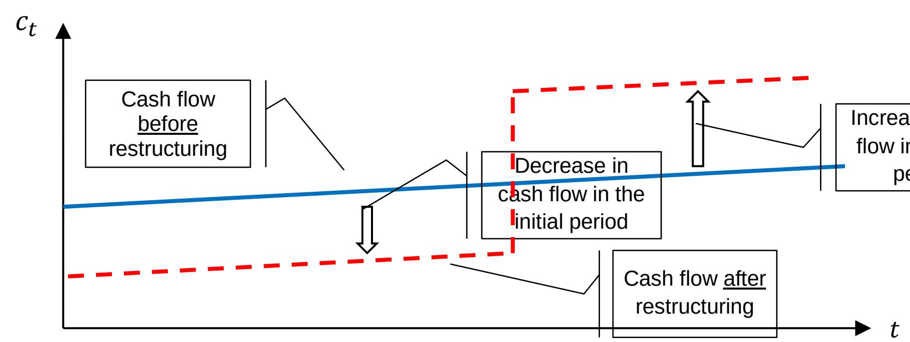


Definition [payment deferral]: Suppose that $c_1, \dots, c_T$ are cash flows before the restructuring and $\tilde{c}_1, \dots, \tilde{c}_T$ are cash flows after the restructuring. Suppose also that $C_t = \sum_{s \le t} c_s$ and $\tilde{C}_t = \sum_{s \le t} \tilde{c}_s$ denote cumulative cash flows before and after loan restructuring. Then the loan restructuring transaction is called to be the loan payment **deferral** transaction if

$$C_t \leq \tilde{C}_t \text{ for each } t \text{ and } C_T = \tilde{C}_T$$

Example of loan deferred payments restructuring transaction is illustrated in the diagram below.

Loan payment deferral type of transaction



Note that interest prepayment / deferral restructuring transactions involve only shift of the payment other backward or forward over time but do not change the cumulative payments over the life of the loan. Note that for both types of transactions the prior to and after restructuring cash flows are assumed to be balances:

$$\sum_{t=1}^{T} c_t = \sum_{t=1}^{T} \tilde{c}_t \tag{d.1}$$

The premium / discount in the loan restructuring transaction is estimated relative to the balanced total cash flows. The prepayment / deferral transactions can benefit both the lender and the borrower depending on the market interest rates and presence of the call/put options.

The prepayment and deferral transactions are **reverse** to each other. That is, if cash flows $c_{2,t}$ are a prepayment transaction with respect to the cash flows $c_{1,t}$, then cash flows $c_{1,t}$ are a deferral transaction relative to the cash flows $c_{2,t}$. As a result, any statement derived for the prepayment type of the transaction can be immediately converted to an equivalent statement for the deferral type of transaction. All statements formulated below are presented in symmetric form for both prepayment and deferral types of loan restructuring. However, some of the proofs are provided only for the prepayment transactions and an equivalent statement for the deferral transaction follows immediately from the inverse relationship between the two types of transactions.

2.2 Examples

The section illustrates different examples of loan cash flow prepayment / deferral structures.

2.2.1 Principal prepayment

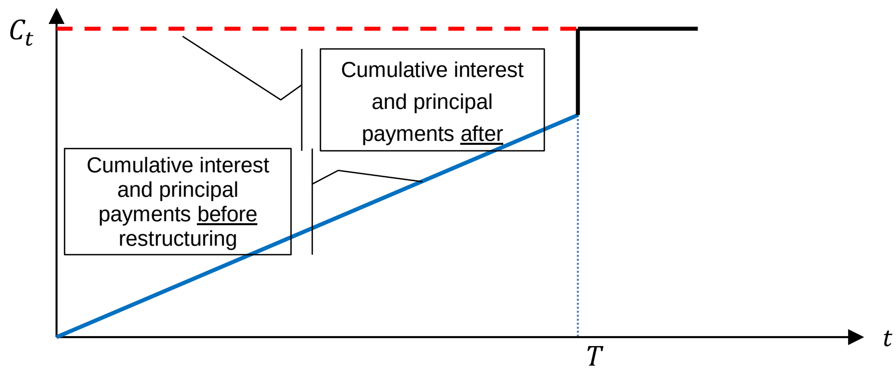
Loan principal can be prepaid either fully or partially. The cumulative cash flows in the two cases are illustrated below.

The loan restructuring is the prepayment of the loan accrued and future interest and principal amount in period $t = 0$. Note that if the future interest is not repaid, then the loan restructuring is not a loan prepayment transaction. The cash flows before and after restructuring are described below.

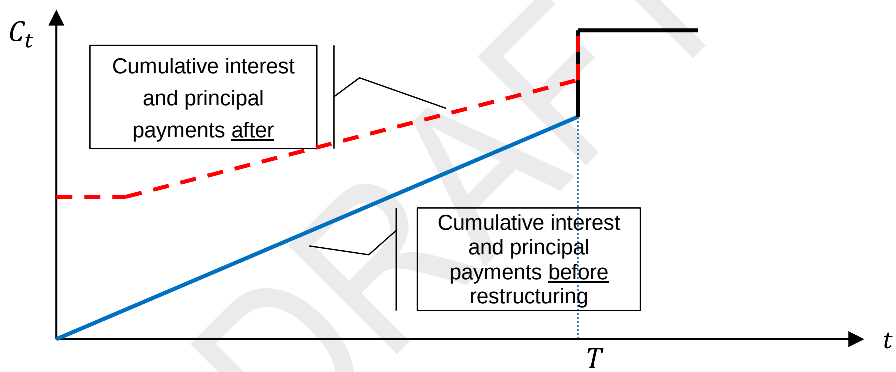
Cash flows before restructuring: $\underbrace{c \times P}_{annual\ interest}, \ldots, c \times P, \underbrace{c \times P + P}_{annual\ interest\ and\ principal}$

Cash flows after restructuring: $\underbrace{T \times c \times P + P}_{prepaid\ interest\ and\ principal}, 0 \ldots, 0$

The cash flows in the full loan repayment restructuring transaction are presented in the diagram below.



The loan principal amount can be prepaid only partially. The diagram with the partial loan prepayment is shown in the diagram below.



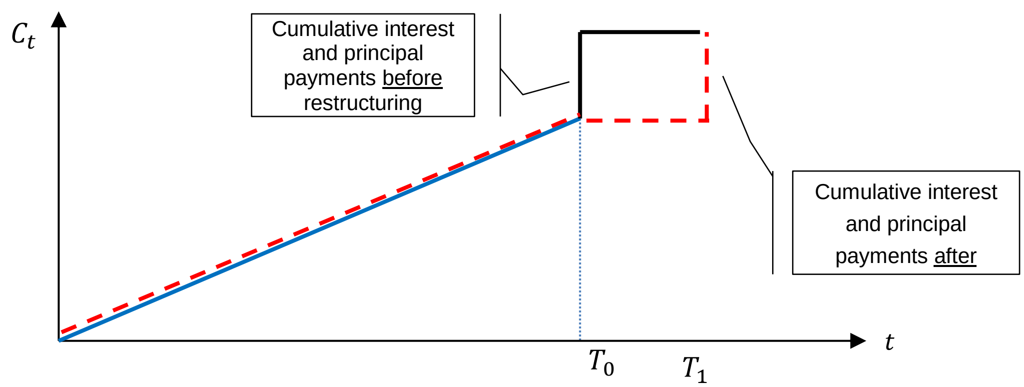
Formally, the partial loan prepayment transaction can be presented as a combination of two transactions. If P is the loan principal and $P = P_1 + P_2$ where P_2 is the prepaid amount, then the two transactions can be described as follows: (i) a loan with principal P_1 with no change in the cash flows; and (ii) the loan with principal P_2 which is fully prepaid.

Note that in order for the total cumulative payments not to change, the borrower must also prepay all forgone future interest together with the principal amount. The penalty can be viewed as a make-whole provision penalty with zero discount rate applied for make-whole provision calculation.

The prepayment without the penalties or a make-whole provisions (or limited penalties) can be viewed as a prepayment type of loan restructuring with the discount provided to the borrower, where the discount is equal to the difference between the prepayment penalty and the penalty under make-whole provision with zero discount rate.

2.2.2 Maturity extension

Maturity extension can be viewed as a principal deferral type of transaction. To ensure that the cash flow balance condition (d.1) holds, it is assumed that coupon payments are zero between the original and extended maturity terms. The cumulative cash flows in the maturity extension transaction is illustrated in the diagram below.



The symbols T_0 and T_1 denote original and extended maturity terms. Maturity extension is a reverse transaction relative to the principal prepayment. If we ignore the cash flows in the $t \in [0, T_0]$ period, which are identical for the two transactions, and assume that period T_0 is the initial period, then the two reverse transactions can be described as follows:

1. Principal prepayment: prepay the original zero-coupon loan with maturity T_1 in period T_0;
2. Maturity extension: extend the maturity of the original loan matured in current period T_0 to period T_1.

2.2.3 Interest prepayment / deferral

This section illustrates three examples: (i) interest prepayment, (ii) interest forgiveness, and (iii) interest deferral. The first example is a type of the prepayment structure. The last two examples are deferral structure types.

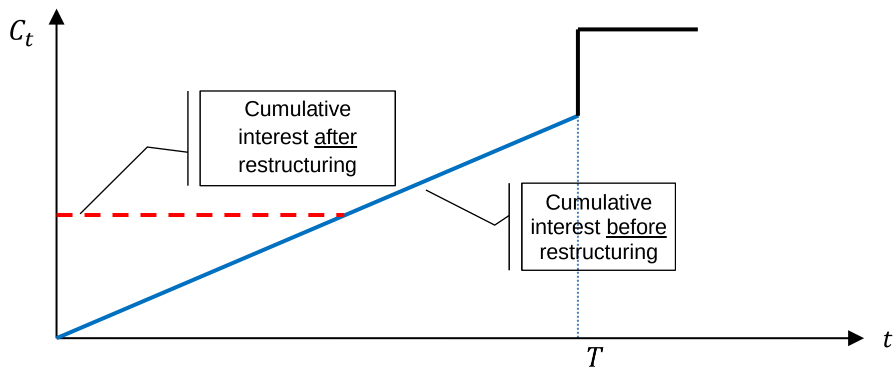
Interest Prepayment

The loan restructuring is the prepayment of the loan accrued interest and part of the loan future interest. The cash flows before and after restructuring are described below.

Cash flows before restructuring: $c \times P, \dots, c \times P, c \times P + P$

Cash flows after restructuring: $\underbrace{t \times c \times P}_{prepaid\ interest}, 0, \dots, 0, c \times P, \dots, c \times P + P$

The cash flows in the interest forgiveness restructuring transaction are presented in the diagram below.



All accrued and some of future interest is prepaid in period $t = 0$. The interest prepayment does not change the principal amount and therefore does not change the future interest and the cumulative principal and interest payments in period T. Therefore, unlike the principal prepayment transaction, the interest prepayment does not require for any penalties or premiums to view at as a prepayment type of loan restructuring.

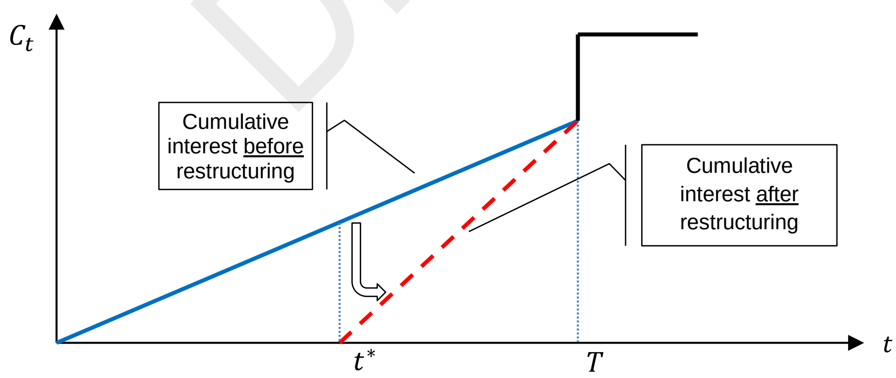
Interest Forgiveness

Interest forgiveness is an example of loan interest deferral restructuring transaction with the following cash flows before and after restructuring.

Cash flows before restructuring: $c \times P, \dots, c \times P, c \times P + P$

Cash flows after restructuring: $\underbrace{0, \dots, 0}_{forgiven\ interest}, \tilde{c} \times P, \dots, \tilde{c} \times P + P$

The cash flows in the interest forgiveness restructuring transaction are presented in the diagram below.



The cumulative interest payments before and after the loan restructuring is shown in the diagram above. In the interest forgiveness restructuring transaction, the borrower does not pay interest (including accrued interest) for a certain specified period of time starting from the loan restructuring effective date. The future interest rate is increased (so that $C_T = \tilde{C}_T$) to compensate lender for the lost revenues from the unpaid interest amount (other alternative form of penalties/premiums may also be considered if the future rate is increased by a given amount for which the $C_T = \tilde{C}_T$ equality does not hold). In the diagram, the form of cumulative interest payments shows that interest forgiveness restructuring transaction is an example of payment deferral transaction. All other terms of the loan are not modified. The analysis can be applied to either fixed or floating rate loan without any conceptual changes.

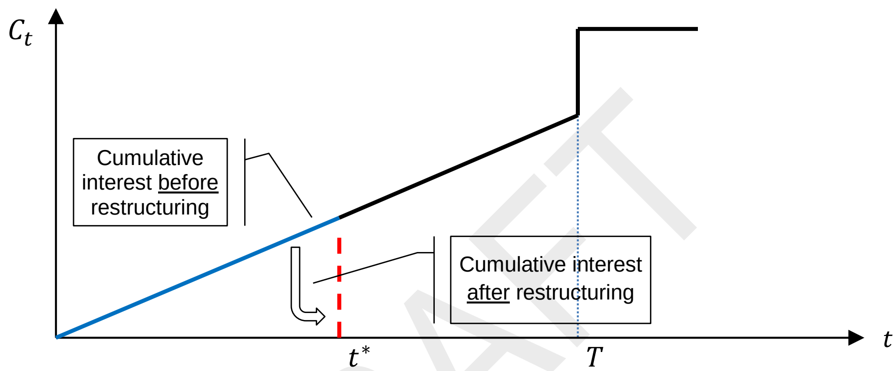
Interest Deferral

Interest deferral is an example of loan interest deferral restructuring transaction with the following cash flows before and after restructuring.

Cash flows before restructuring: $c \times P, \dots, c \times P, c \times P + P$

Cash flows after restructuring: $\underbrace{0, \dots, 0}_{deferral\ period}, \underbrace{t^* \times c \times P}_{deferred\ interest} + c \times P, \dots, c \times P + P$

The cash flows in the interest forgiveness restructuring transaction are presented in the diagram below.



The cumulative interest payments before and after the loan restructuring is shown in the diagram above. The diagram illustrates that the interest deferral restructuring transaction is an example if deferred payments transaction. In the interest deferral restructuring transaction, the borrower does not pay interest (including accrued interest) for a certain specified period of time starting from the loan restructuring effective date and until period t^*. All deferred and accrued interest is repaid after the termination of the interest deferral period. All other terms of the loan are not modified.

Section 3 'Standard' Loan Restructuring Problem

A 'standard' loan restructuring analysis refers to the analysis performed under the following assumptions:

1. The same discount rate is applied to the borrower 's and lender's cash flows. The discount rate is estimated as the refinancing rate of the tested restructured loan transaction.
2. The analysis is performed using 'before tax' loan valuation. The impact of the taxes is not taken into account.

In a standard analysis, there always exists a unique solution[2] of the loan restructuring problem which makes both the borrower and the lender indifferent between the original and restructured loans. There are different alternative ways how to specify the terms of the restructured loan. For example, in the case of interest prepayment / deferral, an addendum in the form of Interest Prepayment Agreement (**IPA**) can be issued to the loan agreement. The IPA can be structured as follows:

1. IPA specifies the premium / discount to the prepaid or deferred interest rate;
2. IPA can specify the period to which the prepaid interest applies. The period could be an extension or shortening of the regular interest payment period.

The application of premium / discount to the prepaid / deferred interest rate depends on the following conditions:

(i) **Terms and conditions** of the loan agreement. A special attention should be given to the prepayment and pay-on-demand options and to penalties applied in the event of interest deferral;

(ii) **Differential** between the interest rate i on the loan (as specified by the loan agreement) and discount rate d estimated for the loan. If $i > d$, then the borrower benefits from the loan principal and/or interest prepayment and, therefore, the interest and principal will be prepaid at premium. Alternatively, if $i < d$, then the lender benefits from the loan principal and/or interest prepayment and, therefore, the interest and principal will be prepaid at discount.

3.1 Scope of 'standard' loan restructuring problem

A scope of 'standard' loan restructuring analysis includes the following steps:

1. Review of the loan agreement terms and conditions;
2. Discussion of the terms and conditions of the restructured loan;
3. Estimation of the credit rating of the borrower and the loan;
4. Estimation of the discount rates applicable to the existing and restructured loans. The discount rate is estimated as the 'before-tax' refinancing rate of the loan;
5. NPV analysis of the original and restructured loan cash flows. The NPV analysis need to take into account existence of the prepayment or pay-on-demand options, prepayment or deferral penalties, and other terms and conditions of the loan.

[2] If the discount rate applicable to the borrower differs from the discount rate applicable to the lender, then (i) either no solution exists which benefits the borrower and the lender in the restructured loan, or (ii) a range of solutions exists which benefits both the lender and the borrower.

3.2 Overview of the analysis

The steps of the analysis are summarized as follows.

1. …

3.3 Approximate calculations

The conceptual idea behind the approximate calculations is to break down the restructuring problem into a sum of basic problems which have an approximate simple intuitive solution. Basic transformations are represented in the prepayment / deferral analysis by shifts of individual payments over time.

Suppose that payment c is shifted from period t_1 to period t_2, where $dt = t_2 - t_1$. Suppose that t_1 and t_2 are consecutive periods in discount factor calculations and discount rate is constant and equal to y so that

$$D_{t_2} = D_{t_1} x \frac{1}{1 + y \times dt}$$

Where D_{t_1} and D_{t_2} are discount factors in periods t_1 and t_2 respectively. Then the change in the loan value from the shift in the payment c is equal to

$$\Delta V = c \times \left(D_{t_2} - D_{t_1}\right) = c \times D_{t_1} \times \frac{y \times dt}{1 + y \times dt} = c \times y \times dt \times D_{t_2} \sim c \times y \times dt$$

The total change in restructured loan value equals to the sum

$$\Delta V^{total} = c \times y \times \sum_k dt^{(k)}$$

where index k represents a basic cash flow transformation.

3.4 Examples

This section illustrates application of approximate calculations in different real-life examples.

3.4.1 Interest prepayment

Example A [**regular prepayment**]. Suppose that a one-year coupon payment is prepaid at the beginning of the year and suppose that coupon rate is fixed at 5% and is paid semi-annually based on 30/360-day count at the last date of each calendar quarter. Suppose that the yield rate is equal to 3%. Estimate approximate reduction in the prepaid coupon rate which would be agreed to by both the borrower and the lender.

Analysis

Two mid-year and end-of-year coupon payments are shifted to the beginning of the year. The lender's gain from the two coupon prepayments is

$$\Delta V = c \times y \times (0.5 + 1.0) \text{ or } c \times y \times 0.75 = 5\% \times 3\% \times 0.75 \sim 10bps$$

per each coupon prepayment. The coupon should be adjusted downward by 10bps on the two prepaid coupon payments.

Example B [**prepayment as compensation for skipped interest**]. Suppose that a first quarter coupon payment was skipped and as a compensation the borrower agrees to prepay the interest for the remaining of the year at the end of quarter two. Suppose also that coupon rate is fixed at 5% and is paid quarterly based on 30/360-day count at the last date of each calendar quarter. Suppose that the yield rate is equal to 3%. Assuming that the penalty on the skipped interest is 1%, estimate the actual approximate penalty that the borrower would pay in addition to interest prepayment.

Analysis

The 1% penalty on a single deferred coupon payment equals to 20% (= 1% / 5%) of the coupon payment. The loss/gain from the deferred and prepaid interest equals approximately to

$$\Delta V = c \times y \times (-0.25 + 0 + 0.25 + 0.5) = 0.5 \times c \times y = 0.5 \times 3\% \times c = 1.5\% \times c$$

where -0.25 represents the cost from the first quarter interest deferral and 0.25 and 0.5 represent the gain from the third and fourth quarter interest prepayment. Note that the prepayment reduces the penalty from 20% to 18.5% (= 20% - 1.5%) of coupon payment or from 1% to 0.925% (= 18.5% x 5%) of principal amount.

Example C [**change in interest frequency**]. Suppose that frequency was changed from semi-annual to annual. Suppose also that coupon rate on the loan is fixed at 5% and the estimated fixed yield rate on the loan is 4%. By how much should the coupon rate be reduced if the coupon frequency is changed from semi-annual to annual.

Analysis

If the frequency is changed from semi-annual to annual, then during each calendar year the first coupon payment is shifted from mid year to the end of the year and the second coupon payment is not changed. The impact on the FMV of the loan is

$$\Delta V = c \times y \times (0.5 + 0) = c \times 2\% = \frac{5\%}{2} \times 2\% = 0.05\%$$

where c is a semi-annual coupon payment. Therefore, the annual coupon rate should be reduced by 5bps (= 10bps / 2).

3.4.2 Validation of interest deferral NPV calculations

The calculations are validated by testing the following statement.

Statement {interest deferral neutrality]. If interest is capitalized and discount rate is fixed and equal to the coupon rate, then interest deferral has zero impact on the change in the loan value.

Suppose that interest was deferred for one period. Note that a general interest deferral transaction can be represented as a sequence of one-period interest deferral transactions.

Under the constant discount rate assumption, the change in value from one-period interest deferral is equal to $\Delta V^a = c \times y \times dt \times D_{t_2}$. If interest is compounded, in period t_1 and is paid in period t_2 then the net present value of additional interest paid on the compounded interest is equal to $\Delta V^b = c \times c \times dt \times D_{t_2}$. If $c = y$ then the cost of interest deferral to the lender equals to the lender's benefit from the additional interest payment.

Note that in the interest deferral transaction the interest rate on the compounded interest must be set equal to the discount rate y to ensure that both the borrower and the lender are indifferent between original loan and the loan with deferred and compounded interest.

QED

3.4.3 Validation of principal amortization NPV calculations

The calculations are validated by testing the following statement.

Statement {principal amortization neutrality]. If discount rate is fixed and equal to the coupon rate, then principal amortization has zero impact on the change in the loan value.

Suppose that $x\%$ of the loan balance was amortized and repaid one period earlier prior to repaying the remainder of the loan balance. Note that a general principal amortization transaction can be represented as a sequence of one-period principal amortization transactions.

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3.5 Summary

Section 4 Discount Differential and Existence of Solution

This section analyzes loan restructuring problem in the case when the discount rate applicable to discount the borrower's and lender's cash flow is different. The focus of the analysis is on deriving the conditions when the solution (or a range of solutions) exists such that both the borrower and the lender are indifferent or better off in the restructured loan.

4.1 Loan restructuring problem

Suppose that c_t represents loan cash flows prior to restructuring and $\tilde{c}_t$ represents loan cash flows post to restructuring. We assume that equation (d.1) holds and any premium / discount is estimated relative to the cash flows $\tilde{c}_t$ normalized by the relative to equation (d.1).[3]

The change in the loan cash flows is assessed from the borrower's and lender's perspective. It must be in the interest of both the borrower and the lender to agree to the loan restructuring conditional on the options available to the borrower and the lender. The borrower's and lender's problem are presented below:

Borrower: $\sum_t c_t^B \times D_t^B \geq \sum_t (\tilde{c}_t^B + c^*) \times D_t^B$

Lender: $\sum_t c_t^L \times D_t^L \leq \sum_t (\tilde{c}_t^L + c^*) \times D_t^L$

Note that the loan interest payments in the borrower's and lender's problem are adjusted by the tax rate and therefore are different from the loan coupon payments. Specifically,

$$c_t^B = (1 - \tau^B) \times c_t \quad and \quad \tilde{c}_t^B = (1 - \tau^B) \times \tilde{c}_t$$

for each borrower loan coupon payment and

$$c_t^L = (1 - \tau^L) \times c_t \quad and \quad \tilde{c}_t^L = (1 - \tilde{\tau}^L) \times \tilde{c}_t$$

for each lender loan coupon payment (before and after restructuring). The borrower's tax rate is assumed to be the same prior to and post restructuring. The lender's tax rate may change after the restructuring.

In general, the discount rates applied by the borrower and lender in the cash flow valuation problem may be different. There are multiple factors that potentially generate loan restructuring benefit for both the lender and the borrower:

1. Movement in market interest / discount rates;
2. Change in the prior to / post restructuring tax rates;
3. Differential between the borrower and the lender discount rates.

The solution of the loan restructuring problem is the range of values c^*, which satisfy both the borrower and the lender constraints.

4.1.1 Tax implications

If tax impact is modelled explicitly as part of a loan restructuring problem, the tax impact on the cash flows and discount rates must be modelled explicitly. Specifically, the '**after-tax**' discount rates must be applied

[3] For example, in the case of principal prepayment, the cash flows $\tilde{c}_t$ include also all future interest payments.

in the valuation analysis, where the distinction between the 'before-tax' and 'after-tax' discount rates is discussed in the 'NPV Analysis' guide. The 'after-tax' discount rates are estimated as described by the equation below.

$$y_t^{after-tax} = y_t \times (1 - \tau) \tag{2.11}$$

where τ is the applicable tax rate. The review of the applicable tax rates must include the following taxes:

1. Income tax in the lender's tax jurisdiction
2. Income tax in the borrower's tax jurisdiction
3. Withholding tax (**WHT**) between the two tax jurisdiction. Note that the WHT is typically paid on intercompany loans only and is zero for third-party loans.[4]

The WHT is typically paid by the borrower on behalf of the lender. The WHT is estimated as a percentage of the all-in (gross) rate and the after-tax interest amount is paid to the lender. The respective relationship between the gross and the net rates is $c^{net} = c^{gross} \times (1 - \tau^{WHT})$ or equivalently $c^{gross} = \frac{c^{net}}{1-\tau^{WHT}}$. The gross rate is included in the loan agreement and is benchmarked in the TP analysis but it is recommended that both gross and net rates were supported by the transfer pricing ranges. The calculation to convert of the net rate into the equivalent gross rate is referred to as to 'gross-up' the interest rate.

4.1.2 Tax neutrality

Tax neutrality of the loan restructuring transaction refers to the case when the taxes do not impact the valuation of the restructuring transaction. In the case of the prepayment / deferral transaction, tax neutrality generally applies in the case of interest expense prepayment / deferral but does not apply in the case of principal prepayment.

Interest rate applies only to the interest income of the lender and interest expense of the borrower. In the case of principal prepayment, the value of the prepaid principal is weighted against the value of the respective change in taxed interest payments. As a result, taxes is one of the factors which effects the value of the prepayment transaction. In the case of interest prepayment / deferral transaction, the necessary condition of the tax neutrality is the condition that the tax rate prior to and after the transaction does bot change.

To summarize, the following two conditions are necessary for tax neutrality principle.

1. Restructuring transaction is interest prepayment / deferral transaction. Alternatively, if restructuring involves principal prepayment, then the lender's and borrower's taxes on interest payments are equal.;
2. The tax rates **do not change** after the restructuring transaction.

4.1.3 Discount rate differential

The default choice for the lender's and borrower's discount rate is the loan refinancing rate, which is estimated as of the loan restructuring valuation date. However, in certain cases it may be argued that the

[4] Note that loan transfer from one tax jurisdiction into another can potentially impact not only the income taxes paid on the loan but also the WHT paid by the counterparties.

borrower or the lender will apply a different discount rate conditional on specific facts applicable to the restructuring transaction.

For example, the borrower may not have a direct access to the capital market and refinance the loan if necessary. Therefore, to replace the loan transaction with different capital structure, the borrower may apply a higher discount rate, such as for example weighted-average cost of capital (**WACC**) rate.

Alternatively, the borrower may have cash which sits idle in the borrower's accounts and does not generate any return. As a result, the borrower will have incentives to use the access of the cash balances to repay the existing debt and may apply a lower discount rate in loan restructuring transaction.

In the next sections, we show the following fact. If tax neutrality applies, there exist parameters of the loan prepayment (deferral) restructuring transaction which is beneficial for both the lender and the borrower whenever the discount rate used by the lender (borrower) is higher than the discount rate of the borrower (lender). The result has a simple interpretation. The borrower and lender will agree to loan prepayment (deferral) transaction whenever the value of the repaid funds is higher (lower) for the lender than for the borrower. If the same discount rate is applied to both the borrower and the lender, then there exists a unique parameter of the loan restructuring transaction which makes both the borrower and the lender indifferent between the original and restructured loans.

4.1.4 Loan refinancing

In some cases, the purpose of the loan restructuring transaction is to refinance the loan to reduce the income taxes. Loan refinancing implies the repayment of the existing loan and, therefore, the transaction is not tax neutral. In addition, the taxes applicable to lender prior and after the loan refinancing are typically different. Since tax optimization is the purpose of the transaction, the fact that tax neutrality principle does not apply to the transaction is expected.

The discount rates for both the lender and the borrower are set to the refinancing rate. The assumption is supported by the fact that existing debt is replaced by the new debt. Therefore, the arguments discussed above, such as the access of cash balances or high cost of capital for the borrower, do not apply.

4.2 Existence of solution

This section analyzes necessary and sufficient conditions for the existence of the prepayment / deferral transaction solution.

4.2.1 Solution existence and discount rate differential (tax-neutral case)

As discussed above, tax neutral case applies to the interest prepayment / deferral transactions assuming that the same tax rate is applied prior to and after the loan restructuring. The necessary and sufficient conditions for the existence of the solution are provided in the following theorem.

Theorem [Solution Existence]: Suppose that tax neutrality applies to the loan restructuring transaction. Then the range of solutions

$$c^* \in [c_{min}, c_{max}]$$

exists whenever one of the two conditions holds:

1. The loan restructuring is the loan **prepayment** type of transaction and $\boldsymbol{y^B} \leq \boldsymbol{y^L}$;
2. The loan restructuring is the loan **payment deferral** type of transaction and $\boldsymbol{y^B} \geq \boldsymbol{y^L}$.

Proof: Under the tax neutrality assumption, the tax rates can be set at zero and the borrower and lender constraints are described as follows.

Borrower: $\sum_t c_t \times D_t^B \geq \sum_t (\tilde{c}_t + c^*) \times D_t^B$

Lender: $\sum_t c_t \times D_t^L \leq \sum_t (\tilde{c}_t + c^*) \times D_t^L$

In the inequalities, the cash flows c_t (prior to transaction) and $\tilde{c}_t$ (after the transaction) are the same from the borrower and the lender perspectives.

The proof is performed in two steps:

1. The theorem is proved for basic "backward cash flow swap" and "forward cash flow swap" restructuring transactions;
2. Each cash flow prepayment transaction is shown to be a composition of "cash flow backward swap" transactions and each cash flow payment deferral transaction is shown to be a composition of "cash flow forward swap" transactions.

The forward and backward cash flow swaps are defined as follows:

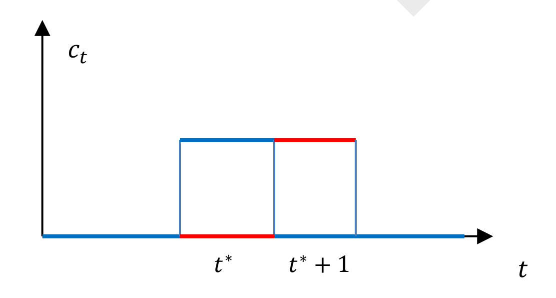
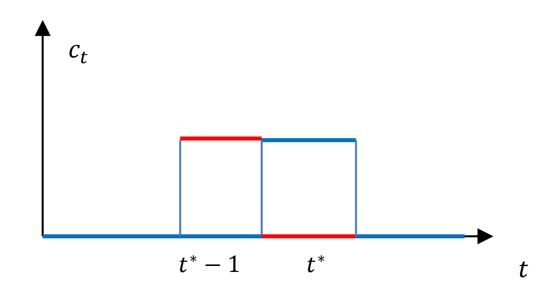
Definition: Suppose that cash flow is represented by a single payment in period t^*: $C = \left(0, \dots, 0, \underbrace{c}_{t^*}, 0, \dots 0\right)$.

Then the forward and backward cash flow swap transactions are described by the following cash flows:

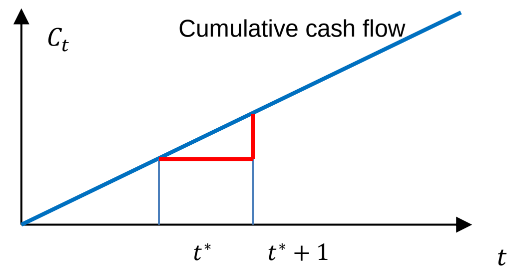
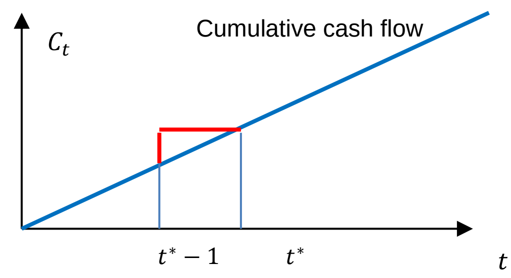
$$C^{fwd\ swap} = \left(0, \dots, 0, \underbrace{0}_{t^*}, \underbrace{c}_{t^*+1}, \dots 0\right)$$

$$C^{bwd\ swap} = \left(0, \dots, \underbrace{c}_{t^*-1}, \underbrace{0}_{t^*}, 0, \dots 0\right)$$

The forward and backward cash flow swap transactions are illustrated in the diagram below.

Forward cash flows swap	Backward cash flow swap
	

When applied to a general cumulative cash flow, the forward and backward cash flow swap transformations look as follows.

<table>
<tr><th>Forward cash flows swap</th><th>Backward cash flow swap</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

The forward swap is an example of payment deferral transaction and backward swap is an example of a prepayment transaction. The Lender's and Borrower's constraints for the forward (backward) swap transaction are described as follows:

Lender: forward: $c \times D_t^L = (c + c^*) \times D_{t+1}^L$ backward: $c \times D_t^L = (c + c^*) \times D_{t-1}^L$

Borrower: forward: $c \times D_t^B \geq (c + c^*) \times D_{t+1}^B$ backward: $c \times D_t^B \geq (c + c^*) \times D_{t-1}^B$

The constraints can be equivalently reduced to the following single constraint:

Forward: $c \times D_t^B \geq c \times \frac{D_t^L}{D_{t+1}^L} \times D_{t+1}^B$ backward: $c \times D_t^B \geq c \times \frac{D_t^L}{D_{t-1}^L} \times D_{t-1}^B$

Or, equivalently,

Forward: $\frac{D_{t+1}^B}{D_{t+1}^L} = \left(\frac{1+y^L}{1+y^B}\right)^{t+1} \leq \frac{D_t^B}{D_t^L} = \left(\frac{1+y^L}{1+y^B}\right)^{t}$ backward: $\frac{D_{t-1}^B}{D_{t-1}^L} = \left(\frac{1+y^L}{1+y^B}\right)^{t-1} \leq \frac{D_t^B}{D_t^L} = \left(\frac{1+y^L}{1+y^B}\right)^{t}$

The constraints are reduced to the following simplified constraints:

Forward (deferral): $\frac{1+y^L}{1+y^B} \leq 1$ or $y^L \leq y^B$; backward (prepayment): $\frac{1+y^L}{1+y^B} \geq 1$ or $y^L \geq y^B$.

To summarize, the range of solutions exists for the forward (backward) cash swap transactions under the following conditions:

Forward swap (**deferral** transaction): $y^B \geq y^L$

Backward swap (**prepayment** transaction): $y^B \leq y^L$

Step two of the theorem is a relatively straightforward statement and the formal proof of the step is not provided.

QED

In the case $y^B = y^L$, the two inequalities that describe the borrower and the lender constraints collapse into a single equation which determines a unique solution c^*:

$$c^* = \frac{\sum_t (c_t - \tilde{c}_t) \times D_t}{\sum_t D_t}$$

The theorem can be further generalized as follows.

Theorem [Solution Properties]: Suppose that $y^B \geq y^L$. Then, if a deferred payment type of loan restructuring (denoted as $\mathfrak{D}$) can be represented as a composition of other two deferred payment restructuring transactions ($\mathfrak{D} = \mathfrak{D}_1 \circ \mathfrak{D}_2$), then the range of solutions for the $\mathfrak{D}$ restructuring is shifted to the right and is broader than the range of solutions for either $\mathfrak{D}_1$ or $\mathfrak{D}_2$ restructuring transactions. (Similar result can be proved for the case of prepayment restructuring and $y^B \leq y^L$ discount rates).

Proof: We prove the theorem for the case when $\mathfrak{D}_2$ is an arbitrary restructuring of payment deferral type and $\mathfrak{D}_1$is a forward cash swap transaction. Suppose that $\{c_s\}$ is original cash flow, $\{\tilde{c}_s\}$ is a cash flow after $\mathfrak{D}_2$ restructuring, and $[c_{min}, c_{max}]$ is a range of solutions for the $\mathfrak{D}_2$ restructuring. Then, by definition,

$$\begin{cases} \sum_s c_s D_s^L = \sum_s (\tilde{c}_s + c_{min}) D_s^L \\ \sum_s c_s D_s^B = \sum_s (\tilde{c}_s + c_{max}) D_s^B \end{cases}$$

Suppose also that restructuring $\mathfrak{D}_1$is describes as a transfer of cash flow $\tilde{c}$ from period t to period $t+1$. Suppose that $\tilde{c}_{min}$ is the minimum adjustment to the lender's constraint to make the lender's indifferent and $\tilde{c}_{max}$ is the maximum adjustment to the borrower's constraint to make the borrower indifferent. The lender's and borrower's constrain can be presented as follows:

$$\begin{cases} (\tilde{c}_t + c_{min}) \times D_t^L + (\tilde{c}_t + c_{min}) \times D_{t+1}^L = (\tilde{c}_t + c_{min} - \tilde{c} + \tilde{c}_{min}) \times D_t^L + (\tilde{c}_t + c_{min} + \tilde{c} + \tilde{c}_{min}) \times D_{t+1}^L \\ (\tilde{c}_t + c_{max}) \times D_t^B + (\tilde{c}_t + c_{max}) \times D_{t+1}^B = (\tilde{c}_t + c_{max} - \tilde{c} + \tilde{c}_{max}) \times D_t^B + (\tilde{c}_t + c_{max} + \tilde{c} + \tilde{c}_{max}) \times D_{t+1}^B \end{cases}$$

The equations can be simplified as follows:

$$\begin{cases} \tilde{c}_{min} = \tilde{c} \times \frac{D_t^L - D_{t+1}^L}{D_t^L + D_{t+1}^L} = \frac{1 - \frac{1}{1+y^L}}{1 + \frac{1}{1+y^L}} = \frac{y^L}{2+y^L} = 1 - \frac{2}{2+y^L} \\ \tilde{c}_{max} = \tilde{c} \times \frac{D_t^B - D_{t+1}^B}{D_t^B + D_{t+1}^B} = \frac{1 - \frac{1}{1+y^B}}{1 + \frac{1}{1+y^B}} = \frac{y^B}{2+y^B} = 1 - \frac{2}{2+y^B} \end{cases}$$

The range of solutions for the $\mathfrak{D}$ restructuring is

$$[c_{min}^* = c_{min} + \tilde{c}_{min}, c_{max}^* = c_{max} + \tilde{c}_{max}]$$

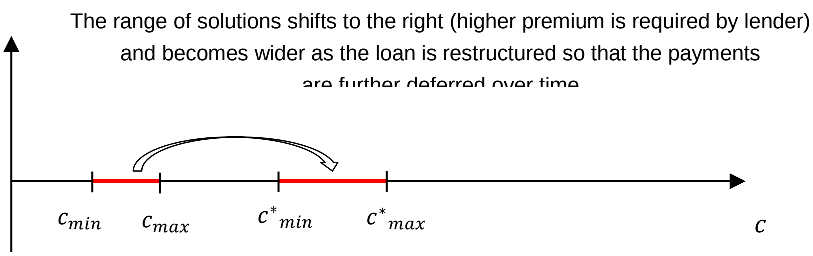
Note that

$$\tilde{c}_{max} \geq \tilde{c}_{min} \geq 0$$

so that the range shifts to the right and the right end of the range shifts more than the left end of the range.

QED

The theorem is illustrated in the diagram below.



The theorem can be directly applied to the case $y^B \leq y^L$.

Section 5 Loan Restructuring Tax Analysis

In this section, we discuss the impact of the tax rates on the existence and properties of the solution. As discussed above, the presence of taxes results in the asymmetric lender and borrower cash flows (and specifically the fact that the cash flow balance equation (d.1) does not hold). The analysis is broken down into two parts:

1. Analysis of solution existence for pre-tax cash flows;
2. Analysis of the tax impact on the pre-tax solution.

In certain cases, such as for example the restructuring of OID notes discussed below, the asymmetry in taxes can be due to two factors: (i) asymmetry in tax rates and asymmetry in the timing of tax payments. The analysis in this case is performed by breaking down and analyzing the impact of each asymmetry separately.

5.1 Interest-bearing loan

5.2 Adjustment for the prepayment (call) / pay-on-demand (put) options

Put and call are adjusted as follows:

1. Adjust interest rate applicable to the loan by adding a call option premium and subtracting put option discount;
2. Add the estimated FMV of the put option discount (cost to the borrower / benefit to the lender) and subtract the FMV of the call option (benefit to the borrower / cost to the lender).

Technically, in the before-tax valuation analysis, the impact of the call option premium and put option discount should be exactly offset by the FMV values of the call and put options so that the refinanced loan is priced at par. In the after-tax analysis this is not necessarily true.

5.3 OID note

In the case of the OID notes, the tax implications for the borrower and the lender are different. Specifically,

1. The borrower can apply tax deductions only in the period when **actual** tax interest is paid. Therefore, the tax deductions are accumulated over the duration of the OID note and are applied for tax deductions only on the OID note maturity date;
2. The lender reports the tax expense on the interest income on a quarterly basis (matching the quarterly frequency of financial reporting). The tax expense is not accumulated and is paid on a regular basis.

Appendix A References